10-3 Transformations of Cube Root Functions

TEKS FOCUS

TEKS (6)(A) Analyze the effect on the graphs of $f(x) = x^3$ and $f(x) = \sqrt[3]{x}$ when $f(x)$ is replaced by $af(x)$, $f(bx)$, $f(x - c)$, and $f(x) + d$ for specific positive and negative real values of a, b, c, and d.

TEKS (1)(F) **Analyze** mathematical relationships to connect and communicate mathematical ideas.

Additional TEKS (1)(D), (2)(A)

VOCABULARY

- **Analyze** – closely examine objects, ideas, or relationships to learn more about their nature

ESSENTIAL UNDERSTANDING

The graph of any cube root function is a transformation of the graph of the cube root parent function $f(x) = \sqrt[3]{x}$.

take note

Key Concept The Cube Root Parent Function

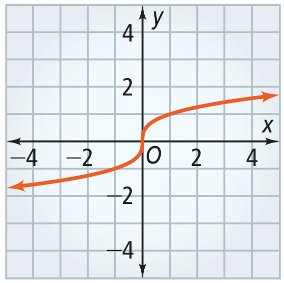
Table

x	$y = \sqrt[3]{x}$
−8	−2
−1	−1
0	0
1	1
8	2

Function

$f(x) = \sqrt[3]{x}$

Graph



take note

Concept Summary Cube Root Function Family

Parent Function $y = \sqrt[3]{x}$

Translation

$y = \sqrt[3]{x} + d$

$d > 0$ shifts up $|d|$ units

$d < 0$ shifts down $|d|$ units

$y = \sqrt[3]{x - c}$

$c > 0$ shifts to the right $|c|$ units

$c < 0$ shifts to the left $|c|$ units

Stretch, Compression, and Reflection

$y = a\sqrt[3]{x}$

$|a| > 1$ vertical stretch

$0 < |a| < 1$ vertical compression (shrink)

$a < 0$ reflection across the x-axis

$y = \sqrt[3]{bx}$

$|b| > 1$ horizontal compression (shrink)

$0 < |b| < 1$ horizontal stretch

$b < 0$ reflection across the y-axis

Problem 1

TEKS Process Standard (1)(D)

Analyzing $y = f(x) + d$ for $f(x) = \sqrt[3]{x}$

What is the graph of the functions $g(x) = \sqrt[3]{x} + 2$ and $h(x) = \sqrt[3]{x} - 2$ on the same set of axes as the parent function $f(x) = \sqrt[3]{x}$?

Graph the parent function $f(x) = \sqrt[3]{x}$.

Each y-value of $g(x) = \sqrt[3]{x} + 2$ is 2 greater than the corresponding y-value of $f(x)$.

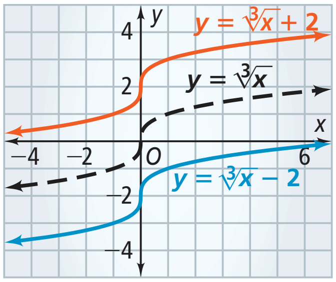
So, $g(x)$ shifts the graph of $f(x)$ up by 2 units.

Each y-value of $h(x) = \sqrt[3]{x} - 2$ is 2 less than the corresponding y-value of $f(x)$.

So, $h(x)$ shifts the graph of $f(x)$ down by 2 units.

Think

How can you check that your graphs are reasonable?
The graphs of $g(x)$ and $h(x)$ should be 4 units apart along any vertical line.



Problem 2

Analyzing $y = f(x - c)$ for $f(x) = \sqrt[3]{x}$

What is the graph of the functions $g(x) = \sqrt[3]{x - 3}$ and $h(x) = \sqrt[3]{x + 3}$ on the same set of axes as the parent function $f(x) = \sqrt[3]{x}$?

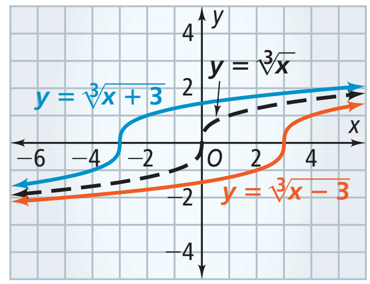
Think

How can you check that your graphs are reasonable?
The graphs of $g(x)$ and $h(x)$ should be 6 units apart along any horizontal line.

Graph the parent function $f(x) = \sqrt[3]{x}$.

A function of the form $f(x) = \sqrt[3]{x - c}$, where $c > 0$, is a horizontal translation of the graph of $f(x)$ by $|c|$ units to the right. So, $g(x) = \sqrt[3]{x - 3}$ shifts the graph of $f(x)$ to the right by 3 units.

A function of the form $f(x) = \sqrt[3]{x - c}$, where $c < 0$, is a horizontal translation of the graph of $f(x)$ by $|c|$ units to the left. So $h(x) = \sqrt[3]{x + 3}$ shifts the graph of $f(x)$ to the left by 3 units.



Problem 3

Analyzing $y = af(x)$ for $f(x) = \sqrt[3]{x}$

What is the graph of the given functions on the same set of axes as the parent function $f(x) = \sqrt[3]{x}$?

A $g(x) = 3\sqrt[3]{x}$, $h(x) = -3\sqrt[3]{x}$

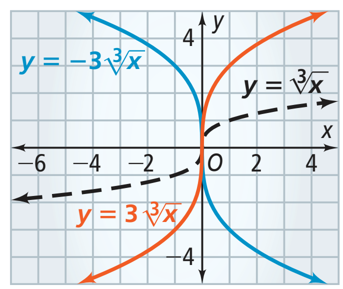
Graph the parent function $f(x) = \sqrt[3]{x}$.

Each y-value of $g(x) = 3\sqrt[3]{x}$ is 3 times the corresponding y-value of the parent function. So, $g(x) = 3\sqrt[3]{x}$ stretches the graph of $f(x)$ by the factor 3.

The -3 in $h(x) = -3\sqrt[3]{x}$ reflects the graph of $f(x)$ across the x-axis and stretches it by the factor 3.

Think

How are the graphs of $g(x)$ and $h(x)$ related?
The graphs are reflections of each other in the x-axis.

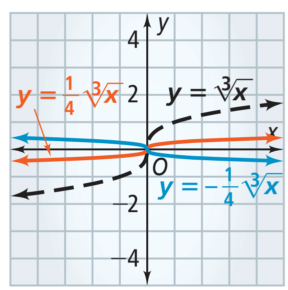


B $g(x) = \frac{1}{4}\sqrt[3]{x}$, $h(x) = -\frac{1}{4}\sqrt[3]{x}$

Graph the parent function $f(x) = \sqrt[3]{x}$.

Each y-value of $g(x) = \frac{1}{4}\sqrt[3]{x}$ is one-fourth the corresponding y-value of the parent function. So, $g(x) = \frac{1}{4}\sqrt[3]{x}$ compresses the graph of $f(x)$ by the factor $\frac{1}{4}$.

The $-\frac{1}{4}$ in $h(x) = -\frac{1}{4}\sqrt[3]{x}$ reflects the graph of $f(x)$ across the x-axis and compresses it by the factor $\frac{1}{4}$.



Problem 4

Analyzing $y = f(bx)$ for $f(x) = \sqrt[3]{x}$

What is the graph of the given functions on the same set of axes as the parent function $f(x) = \sqrt[3]{x}$?

Think

What is the amount of the horizontal stretch?

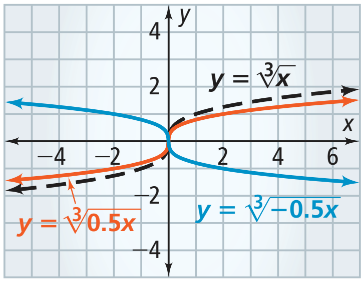
Since $b = 0.5$ and $0 < |b| < 1$, $f(x)$ is horizontally stretched. Notice that $f(x) = 2$ when $x = 8$, but $g(x) = 2$ when $x = 16$. So, $g(x)$ is a horizontal stretch of $f(x)$ by a factor of 2.

A $g(x) = \sqrt[3]{0.5x}$, $h(x) = \sqrt[3]{-0.5x}$

Graph the parent function $f(x) = \sqrt[3]{x}$.

A function of the form $f(x) = \sqrt[3]{bx}$ where $0 < |b| < 1$ is a horizontal stretch of the graph of $f(x)$. So, $g(x) = \sqrt[3]{0.5x}$ stretches the graph of $f(x)$ horizontally.

The -0.5 in $h(x) = \sqrt[3]{-0.5x}$ reflects the graph of $f(x)$ across the y-axis and stretches it horizontally.

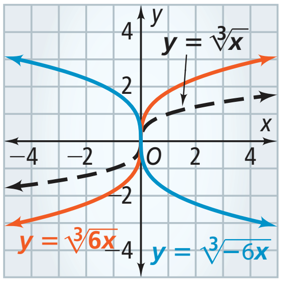


B $g(x) = \sqrt[3]{6x}$, $h(x) = \sqrt[3]{-6x}$

Graph the parent function $f(x) = \sqrt[3]{x}$.

A function of the form $f(x) = \sqrt[3]{bx}$, where $|b| > 1$, is a horizontal compression of the graph of $f(x)$. So, $g(x) = \sqrt[3]{6x}$ compresses the graph of $f(x)$ horizontally.

The -6 in $h(x) = \sqrt[3]{-6x}$ reflects the graph of $f(x)$ across the y-axis and compresses it horizontally.



Problem 5

TEKS Process Standard (1)(F)

Graphing a Cube Root Function

The function $y = 2\sqrt[3]{x + 1} + 4$ models the monthly profit y, in thousands of dollars, x months after a café opens for business. What is the graph of the function?

Plan

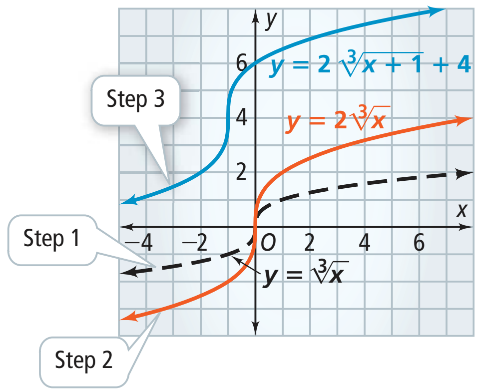
How is $y = a\sqrt[3]{x - c} + d$ related to its parent function?
The factor a vertically stretches or compresses the parent function while c and d translate it horizontally and vertically.

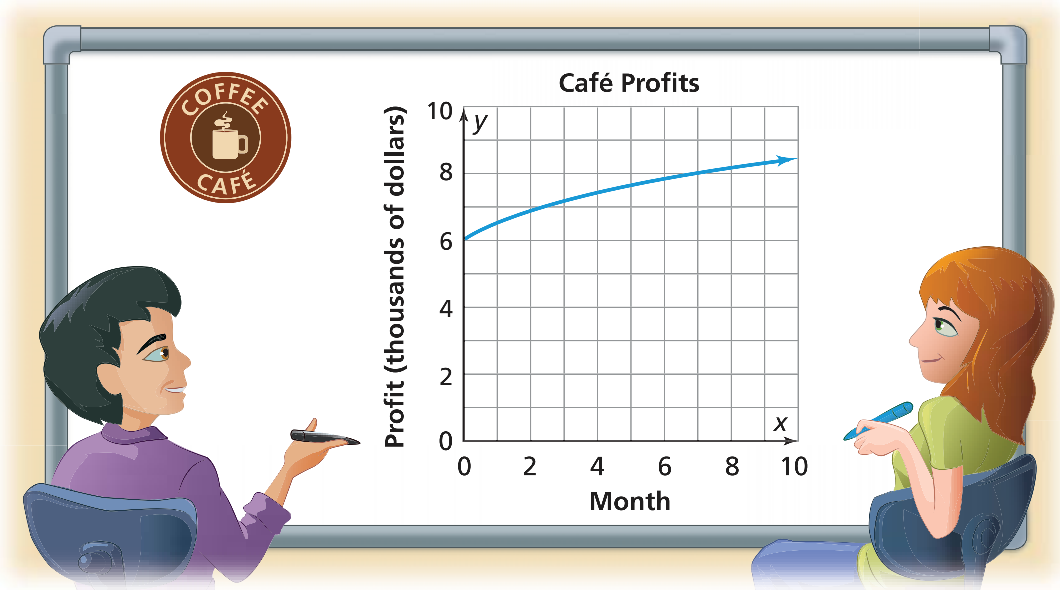
Step 1 Graph the parent function $y = \sqrt[3]{x}$.

Step 2 Multiply the y-coordinates by 2. This stretches the graph vertically.

Step 3 Translate the graph 1 unit to the left and 4 units up.

Step 4 In this situation, the domain is $x \geq 0$. Redraw the graph in the first quadrant only.





PRACTICE and APPLICATION EXERCISES

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Graph each transformation of the parent function $f(x) = \sqrt[3]{x}$ on the same set of axes as the parent function. Analyze the effect of the transformation on the graph of the parent function.

1. $g(x) = \sqrt[3]{x} + 4$

2. $g(x) = \sqrt[3]{-3x}$

3. $g(x) = -2\sqrt[3]{x}$

4. $g(x) = \sqrt[3]{x} - 1$

5. $g(x) = \sqrt[3]{\frac{1}{4}x}$

6. $g(x) = \frac{1}{3}\sqrt[3]{x}$

7. $g(x) = \sqrt[3]{-\frac{1}{10}x}$

8. $g(x) = \sqrt[3]{x + 3.5}$

9. $g(x) = \sqrt[3]{x - 1.5}$

10. The function $y = -2\sqrt[3]{x + 4} + 8$ models the price y of one share of a company's stock, in dollars, x months after the stock became available.

a. Describe a sequence of transformations you can use to graph the function if you start with the graph of the parent function $f(x) = \sqrt[3]{x}$. Graph the function.

b. What does the graph tell you about the price of the company's stock?

c. Do you think the function is a good model of the stock's price for any number of months? Explain.

11. Explain Mathematical Ideas (1)(G) Your friend said the graph of $g(x) = \sqrt[3]{-x}$ is a reflection of the graph of the parent function $f(x) = \sqrt[3]{x}$ in the x-axis. You said the graph of $g(x)$ is a reflection of the graph of the parent function in the y-axis. Who is correct? Explain.

The graph of $g(x)$ can be obtained from the graph of the parent function $f(x) = \sqrt[3]{x}$ by using the given transformations. Write an equation for the function $g(x)$.

12. Reflect the graph in the x-axis, then translate it 2 units right.

13. Vertically compress the graph by a factor of $\frac{1}{3}$, then translate it 4 units left and 1 unit up.

14. Vertically stretch the graph by a factor of 6, then translate it 1 unit right and 7 units up.

15. Horizontally stretch the graph by a factor of 2, then translate it 2 units down.

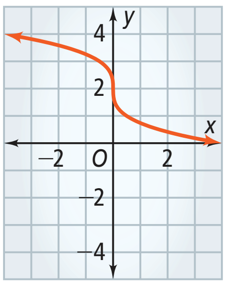
Match each function with the correct graph.

16. $y = \sqrt[3]{-2x} + 2$

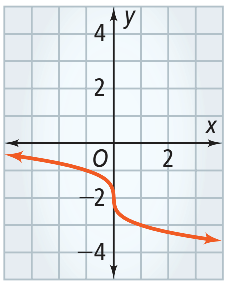
17. $y = 2\sqrt[3]{x} + 2$

18. $y = -\sqrt[3]{x} - 2$

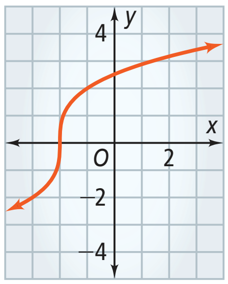
A.



B.



C.



19. **Analyze Mathematical Relationships (1)(F)** The function $y = 0.5\sqrt[3]{x - 2} + 3$ models the annual revenue y of a software company, in millions of dollars, x years after 2000.

a. Describe a sequence of transformations you can use to graph the function if you start with the graph of the parent function $y = \sqrt[3]{x}$.

b. Graph the function.

c. What does the graph tell you about the company's annual revenue since 2000?

Determine whether each statement is always, sometimes, or never true.

20. For a not equal to zero, the graph of $f(x) = a\sqrt[3]{x}$ rises as x increases.

21. The graph of $f(x) = \sqrt[3]{x} + d$ has 180° rotational symmetry around a point on the y-axis.

22. You can draw the graph of $g(x) = \sqrt[3]{x - c}$ by translating the graph of the parent function $f(x) = \sqrt[3]{x}$.

23. The graph of $g(x) = -\sqrt[3]{x} - 1$ passes through the point $(a, 0)$, where $a > 0$.

24. For real numbers m and n, with $m \neq n$, the graph of $f(x) = \sqrt[3]{x - m}$ intersects the graph of $g(x) = \sqrt[3]{x - n}$.

TEXAS Test Practice

25. Which function has a graph that is not a translation of the graph of the parent function $f(x) = \sqrt[3]{x}$?

A. $g(x) = \sqrt[3]{x - 3.7}$

B. $g(x) = \sqrt[3]{x} + 3.7$

C. $g(x) = \sqrt[3]{3.7x}$

D. $g(x) = \sqrt[3]{x + 3.7}$

26. Which function has a graph that intersects the negative x-axis?

F. $f(x) = \sqrt[3]{x} + 8$

G. $f(x) = -\sqrt[3]{x} + 8$

H. $f(x) = \sqrt[3]{8x}$

J. $f(x) = \sqrt[3]{x - 8}$

27. You graph the function $f(x) = \sqrt[3]{x}$. Then you reflect the graph across the x-axis, stretch the graph vertically by a factor of 2, and then translate the graph 2 units to the right. Which of the following is an equation for the resulting graph?

A. $y = -2\sqrt[3]{x + 2}$

B. $y = -2\sqrt[3]{x - 2}$

C. $y = -2\sqrt[3]{x} - 2$

D. $y = \sqrt[3]{-2x} + 2$

28. Explain how the graph of $g(x) = 4\sqrt[3]{x - 1}$ is related to the graph of $h(x) = 4\sqrt[3]{x + 4}$.